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Name of College :- S.S. College Jehanabad

Deptt - Mathematics

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Topic - System of circles, Radical Axis and Co-Axial circles (Analytical Geometry of two Dimension) Class - B.Sc I (Sub)

Time —

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Radical Axis :→

The Radical Axis of two circles is the locus of a point which moves so that the lengths of the Tangents drawn from it to the two circles are equal.

Let the equation of two Circles be

$$x^2+y^2+2gx+2fy+C=0 \quad \text{——— (I)}$$

and. $$x^2+y^2+2g_1x+2f_1y+C_1=0 \quad \text{——— (II)}$$

Let $P(h,k)$ be a point from which the tangents to (I) and (II) are equal

therefore

$$h^2+k^2+2gh+2fk+C = h^2+k^2+2g_1h+2f_1k+C_1$$

$$\Rightarrow 2(g-g_1)h+2(f-f_1)k+(C-C_1)=0$$

Therefore, the locus of (h,k) is a st. line

$$2(g-g_1)x+2(f-f_1)y+(C-C_1)=0$$

which. It is the equation of Radical Axis.

Remarks :→ The Radical Axis of two Circles is their common chord.

The Radical Axis of the circles
$x^2+y^2=2x$
and $2x^2+2y^2-3y=5$ is

$(2x^2+2y^2-4x)-(2x^2+2y^2-3y-5)=0$

$\Rightarrow -4x+3y+5=0$

$\Rightarrow 4x-3y-5=0$

$\Rightarrow 4x-3y=5$

Remarks :- Common chord is real But the meets of the circles may be imaginary.

Theorem Relating to Radical Axis —

The Radical Axis of two circles is a st. line $\perp$ to the line joining the centres —

Let equation of two circles be

$x^2+y^2+2gx+2fy+c=0$ — (I)

and $x^2+y^2+2g_1x+2f_1y+c_1=0$ — (II)

then the co-ordinate of centres of (I) and (II) are $(-g,-f)$ and $(-g_1,-f_1)$ respectively

$\therefore$ Slope of the line joining their centres is $\dfrac{f-f_1}{g-g_1}$

But the slope of the radical axis

$2(g-g_1)x+2(f-f_1)y+c-c_1=0$

is Given by $-\dfrac{g-g_1}{f-f_1}$

But as the product of slope is -1, the lines are right angles.

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Show that the radical axis of three circles taken in pairs are concurrent.

Let the equations to the three circles be

$S_1 = 0$ —— (I)

$S_2 = 0$ —— (II)

$S_3 = 0$ —— (III)

Radical axis of (II) and (III) is

$S_2 - S_3 = 0$ —— (IV)

the Radical Axis of (III) and (I) is

$S_3 - S_1 = 0$ (V)

the Radical Axis of (I) and (II) is

$S_1 - S_2 = 0$ (VI)

Adding (IV) (V) and (VI), the sum identically vanishes.

Therefore the equation (IV), (V) and (VI) hold simultaneously.

i.e Radical axes meet in a point.

Art. 3 Find the locus of the centre of a circle cutting two circles orthogonally

Let the ~~two~~ equation of the two circles be

$$x^2+y^2+2g_1x+2f_1y+c_1=0 \quad \text{—— (I)}$$

$$x^2+y^2+2g_2x+2f_2y+c_2=0 \quad \text{—— (II)}$$

Let the equation of the circle which cuts them orthogonally

Le
$$x^2+y^2+2gx+2fy+c=0 \quad \text{--- (III)}$$
Centre of (III) is $(-g,-f)$

Since (III) intersect (I) orthogonally

$$\Rightarrow 2gg_1+2ff_1=c+c_1 \quad \text{--- (IV)}$$

Similarly
$$2gg_2+2ff_2=c+c_2 \quad \text{--- (V)}$$

(IV) — (V)

$$2g(g_1-g_2)+2f(f_1-f_2)=c_1-c_2$$

Therefore the locus of the centre $(-g,-f)$ is

$$2(-x)(g_1-g_2)+2(f_1-f_2)(-y)=c_1-c_2$$
$$\Rightarrow 2x(g_1-g_2)+2y(f_1-f_2)+c_1-c_2=0$$

Which is the equation of the Radical Axis of the two circles.

Therefore the locus of the centre of the circle which cuts two given circles orthogonally is a st. line and this locus always passes through two fixed points.

Remarks: → If a circle intersects two circles orthogonally, the Radical Axis of the two circles passes through the centre of the circle.

* The General Equation of circles passing through common points of two given circles —

Let $S = x^2+y^2+2gx+2fy+c=0$
and $S_1 = x^2+y^2+2g_1x+2f_1y+c_1=0$
be the two given circles

Consider the equation
$$S+KS_1=0$$
for different values of K, it represent a circle, Since there is no term xy in it and the co-efficient of x^2 is equal to the co-efficient of y^2

Also it passes through the common points of
$$S=0$$
$$S_1=0$$
Hence for different values of K
$S+KS_1=0$ represents a system of circles passing through the common points of
$$S=0$$
$$S_1=0$$
and hence co-axial with the circles
$$S=0$$
$$S_1=0$$